A Statistical Model for 1800 MHz Spectrum Price in India – Part II.

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[Ver.2]

Abstract

The Price of spectrum in a Spectrum auction is a complex function of many factors like physical characteristic of the frequency band, socio-economic factors, the policy and regulation of the telecom field in a country and many more. The first exercise for the auctioneer is to get the Reserve Price right so as to spur competition and economic activity in the bidding process. Econometrics approach is one of the fields that integrates mathematics, statistics, and economic theory together and offer a rational technique for the estimation of reserve price. Econometric modelling requires the real data from the past in order to conceive a mathematical model. The model can explain a linear association between dependent variables and various influencing factors in the form of independent variables. In this paper, with a variety of estimated price points from the previous auction background of 2012, 2014, 2015, 2016 and 2021 auctions as independent variables and the actual winning price in these auction as the dependent variable, a moderate sized dataset is carved out and a model is developed using econometrics modelling that can be used for estimation of the predicted value of price of 1800 MHz spectrum in India.

The weak bidding activity in all the auctions that were held, except that in the year 2014, has not helped in building a repertoire of auction winning prices that are clearly distinct from the reserve price, but are prices that often lurks in the shadow of the reserve price. In other words, the estimated price from the regression model can be associated with either the auction price or the reserve price since these two are not different in the dataset gathered. The inadvertent finding from the linear regression is that it has produced a model that relatively can be used to evaluate the reserve prices from the different price metrics recommended by TRAI for auction while at the same time predicting a winning price as well.

Contents: -

1. Introduction.

In an earlier paper dated 23rd May 2020 (*Ref 1), a statistical model was developed by this author to calculate the spectrum price of 1800 MHz spectrum in India. The model was based on the data collected from 4 spectrum auctions held in India viz. in 2012, 2014, 2015 and 2016. The paper focussed on the metrics adopted by TRAI for the calculation of the reserve price of spectrum for the 1800 MHz band. An association between the Winning price in an auction as the dependent variable and the TRAI metrics and some macro-demographic estimators as the independent variables was established by an econometrics model using linear regression.

It may be recalled from that paper that the reserve price fixed by TRAI is selected as the starting point to determine the Government approved reserve price which will be deployed by Department of Telecom (DoT), the body who conducts the spectrum auction. An internal process is used by DoT to derive this reserve price of the spectrum for each LSA. This price is later notified as the Reserve Price for conducting the auction in the Notice Inviting Auction (NIA), a document that list all information and conditions related to auction.

In the 2020 paper, it was established that the winning price of spectrum in the auction was decided by three explanatory variables namely the price decided by discounted cash flow (DCF) method by TRAI, the population in an LSA and the GSDP/Capita of that LSA. The OLS regression was conducted with 80 data points assimilated from the four spectrum auctions to find the association between winning price and the selected independent variables. The regression yielded the following linear association: -

Log (Winning Auction Price) = 5.4287 + 0.4476 X Log (Population in numbers)

+ 0.3795 X Log (DCF in Rs Crores per MHz)

+ 0.4588 X GSDP/capita in Rs Lakhs Per population.

More data from auction of 2021: - After auction of 2016, the next auction was held only in March 2021. In this auction multiple frequency bands were offered for auction including the 1800 MHz spectrum. The quantum of spectrum available for auction varied from LSA to LSA, the lowest is 1MHz in Kolkata LSA and 20.6MHz offered in

Tamilnadu LSA. TRAI submitted its recommendation dated 1st August 2018 to DoT which contained all the valuation metrics used by the TRAI to determine the Reserve Price for all Spectrum bands including 1800 MHz. Bound by the convention, TRAI used a probabilistic average of different metrics in deciding the final TRAI reserve price. The additional information about the TRAI recommendations and the subsequent auction held in March 2021 are fresh input to explore the correlation.

Armed with this additional input from the latest auction, this paper is an attempt to review and relook at the association established between the response variable (Winning Price) and other explanatory variables. At a fundamental level, it is assumed to have the existence of linear association between the independent variables and the dependent variable. The same OLS regression method will be used to find if the derived association will hold ground with the additional information generated from the latest auction.

2. **Research question recalled**: The research question is reproduced from the previous paper updating with information about the 2021 auction as:

"To find the association, if any, between the Auction price of 1800 MHz spectrum in India in terms of the TRAI estimated price metrics and other demographic, macro economical and telecom predictors available for the years 2012, 2014, 2015, 2016 and 2021 using linear regression method and if so, predict the auction price of the same spectrum for the new (future) data"

3. New data and new input variables:

From the recommendation paper issued by TRAI dated, the price of 1800 MHz spectrum tabulated by different valuation metrics is extracted. The DoT fixed the reserve price through its internal process from the 'TRAI fixed RP'. The 'Govt fixed RP' will be derived by DoT by internal process. After the auction was completed, the winning price of the 1800 MHz band for the different LSAs was taken from the DoT website. This 'Winning Auction Price' along with the 'GOVT fixed RP' for each LSA are merged with the table extracted from TRAI recommendation to generate the following table with 7 different price columns:

(All prices are in Rs. Crores per MHz spectrum.)

Service Area	Producer Surplus	Production Function	Revenue Surplus (DCF)	Previous auction Price	Trai fixed RP	Govt fixed RP	Winning Auction Price
Andhra Pradesh	147.62	91.03	161.96	278.58	279	279	279
Assam	47.63	26.62	38.36	45.86	46	46	46
Bihar	244.28	75.73	66.53	71.08	88	88	88
Delhi	336.03	206.13	127.76	457.42	457	457	457
Gujarat	208.18	73.33	84.23	272.85	273	273	273
Haryana	47.22	22.8	26.85	56.52	57	57	57
HP	16	12.73	12.24	18.34	18	18	18
J&K	32.37	22.78	21.6	14.9	15	17	17
Karnataka	162.55	95.56	152.45	185	109	109	109
Kerala	101.58	46.93	125.25	95.15	95	95	95
Kolkata	75.07	36.41	28.24	173.28	173	173	173
Maharashtra	167.09	64.25	71.42	95.15	95	95	95
MP	237.52	105.07	145.74	364.56	365	365	365
Mumbai	260.52	139.15	93.06	560.82	561	561	561
North East	37.68	14.5	23.76	12.61	13	17	17
Orissa	46.5	30.97	23.85	33.1	27	27	27
Punjab	163.54	39.73	68.96	88.27	88	88	88
Rajasthan	153.34	65.97	64.24	105.36	105	105	Not sold
TN	131.38	93.94	148.26	93.94	100	100	100
UP(E)	218.56	80	81.61	152.64	153	153	153
UP(W)	109.73	60.03	50.96	114.99	115	115	115
WB	59.55	46.53	35.81	52.73	53	53	53

The three valuation metrics used by TRAI that are important for this paper are the price obtained by 'Production function' method, 'Producer surplus' method and 'Discounted cash flow' (DCF) method. The previous auction price of 2016 auction was also used as another independent variable along with the three TRAI metrics. These four different prices were taken from the TRAI recommendations dated 1st August 2018 for this paper. However in the recommendation, the corresponding values for Karnataka and Orissa in the 'previous auction price' column were not available as the spectrum for these LSA s were not sold in the 2016 auction. In order to fill the missing values, the price determined in the 2015 auction was substituted. Another point to note from the above table, that out of the 22 LSA s where the spectrum was offered for auction, it was sold in all LSA s except Rajasthan LSA where no spectrum was sold. The Winning price is the response variable and cannot afford to have a missing value for response variable. So, the entry for the Rajasthan LSA of 2021 will be dropped from the data effectively reducing the data points from 22 to 21.

The next task is to fill up the corresponding telecom data, demographic data and the GSDP data for each LSA for this year. The said data for the year 2020 is available from the same data repository through which it was sourced in the previous paper. Since TRAI and DoT has formulated the price metrics and reserve price during the period 2018-2020, by the same logic the data for the remaining variables were also taken for the year 2020 to fit into the new data table. Now appending these 21 data points to the original dataset will result in augmenting the dataset to 101 records.

Extra input variables: - Two new variables are introduced in this paper. The quantum of spectrum auctioned in each of the LSA which is a direct indicator of the demand of spectrum in that LSA. Another indicator for the vitality of an LSA in telecom terms is the Aggregate Gross Revenue (AGR) of that LSA which is sourced from the combined revenue of all Telecom Service Providers (TSP) in an LSA. The AGR is expected to influence the purchasing power of a TSP which in turn will dictate the final auction price. Intuitively, these two variables are expected to influence the winning price and therefore considered as potential input variables.

The spectrum details are taken from the DoT website which lists the auction outcome for each auction where the details are available for each LSA and for

different frequency bands. This data was directly fetched. The AGR details are extracted from the financial indicators of telecom service providers given in the TRAI report. The quarter-yearly data was collected, processed and compiled to convert it into yearly AGR data.

There is only one missing data in 2021 in the case of Rajasthan circle in which spectrum was offered but not sold. As stated earlier, this record was dropped from the 2021 data. Along with the existing 80 data points for the years 2012, 2014, 2015 and 2016, by adding the 2021 records, the total data points now come to 101.

4. **Complete list of variables**: The list of dependent and dependent variables are reproduced here: -

The following is the final list of explanatory variables and the response variable used in the study:

a) Response Variable: Auction Price (AP) in Rs. Crores per MHz

b) Explanatory Variables:

1. The price of spectrum calculated by Production function method in Rs. Crores per MHz
2. The price of spectrum calculated by Producer Surplus method in Rs. Crores per MHz
3. The price of spectrum calculated by DCF method in Rs. Crores per MHz
4. The price of spectrum calculated based on the previous auction price method in Rs. Crores per MHz
5. The price of spectrum fixed as the final Reserve Price in Rs. Crores per MHz
6. GSDP per capita in Lakhs of Rupees per population.
7. Population of each LSA.
8. ARPU expressed in Rs. /Subscriber/month
9. Minutes of Use (MOU) expressing the mobile telephone traffic in Minutes.
10. No of Mobile subscribers expressed in Millions.
11. Tele density expressed as the number of mobile connections per 100 people.
12. Spectrum sold in the corresponding auction in MHz.
13. The AGR in Rs.Crores.
14. The LSA (categorical variable)

15. Class of the LSA (categorical variable)
16. Year of auction (categorical variable)

The above variables are collected for each of the 22 Licensed Telecom Service Areas. The independent variables are quantitative (continuous) except the last 3 which are categorical and the response variable also is quantitative.

5. Exploratory Data Analysis(EDA): -

(i) Box plots: -The box plot shows the distribution of data for the three TRAI metrics and the response variable against the 5 auction years. The response variable is uniformly distributed across all the auction years with the median value between 50 to 80 Rs Crores. There are few outliers also beyond Rs.300 Crores. As for the three TRAI metrics, there is no consistency among the three for obvious reasons since the principle of calculation in each metrics follow different methodology. Among the three, the DCF method has a more pronouncing effect while comparing the plots for all the 5 years.

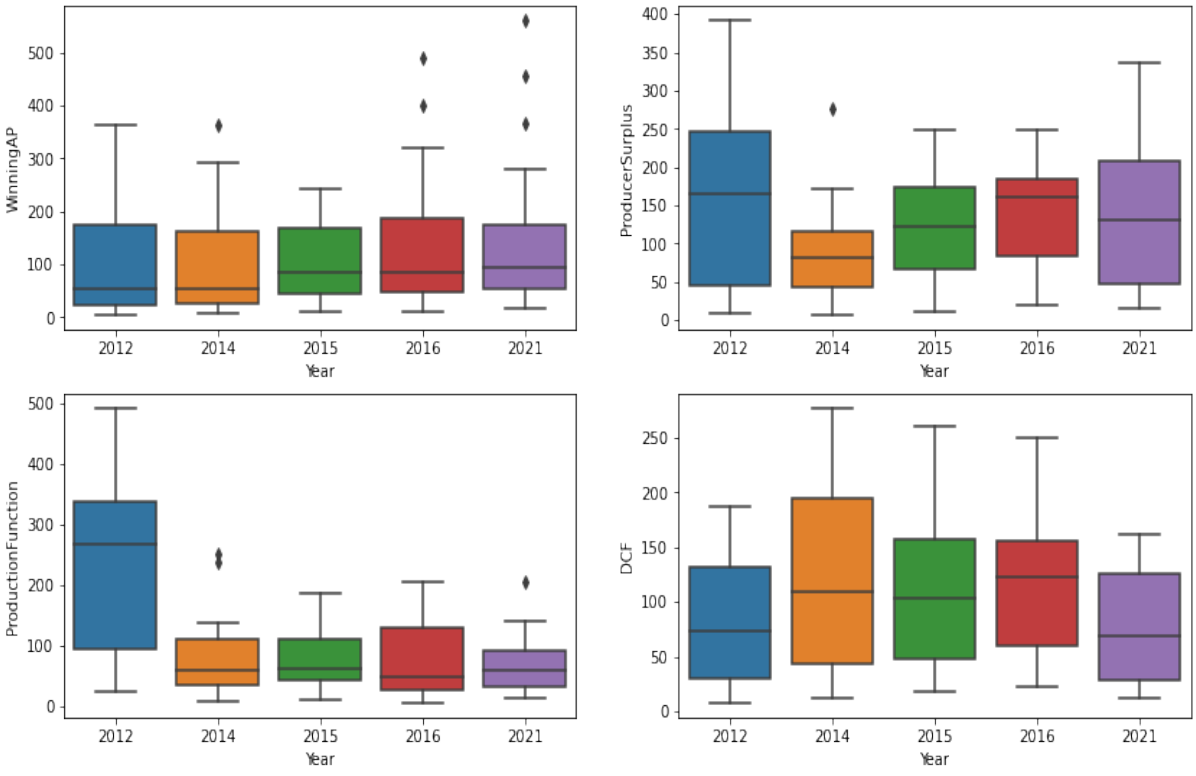


Fig-1

The second plot (fig.2) is the distribution of the same variables compared among the different LSA category- Metro, Class A, Class B and Class C. The median value of the Winning price is substantially higher for Metro and Class ‘A’ LSA s compared to the Class ‘B’ and Class ‘C’ LSAs. The variability of the Winning price is reflected more in the case of Metro whereas in the case of Class B and Class C, the data window emaciates to a very narrow variance. And among the TRAI metrics, the DCF and Producer Surplus are found to have consistent variance among all the categories of LSA unlike Production Function metrics in which case, the influence is more prominent in the case of Metro and Class A LSA s.

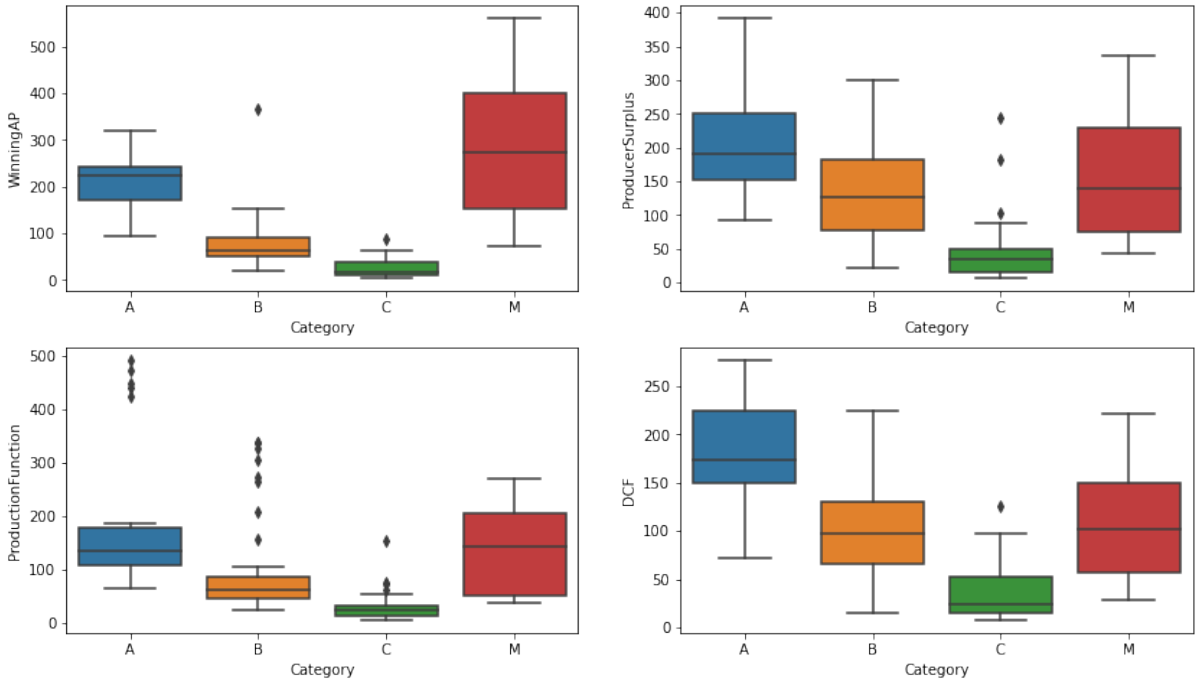


Fig.2

(ii)The Histograms: -

The histograms (fig-3) show the skewed distribution of data. The logarithmic transformation is used to reduce this skewedness for all right skewed variables. The response variable also is skewed right which is again transformed using the Log conversion. The price variables after the transformation look like as that in the Fig-4:

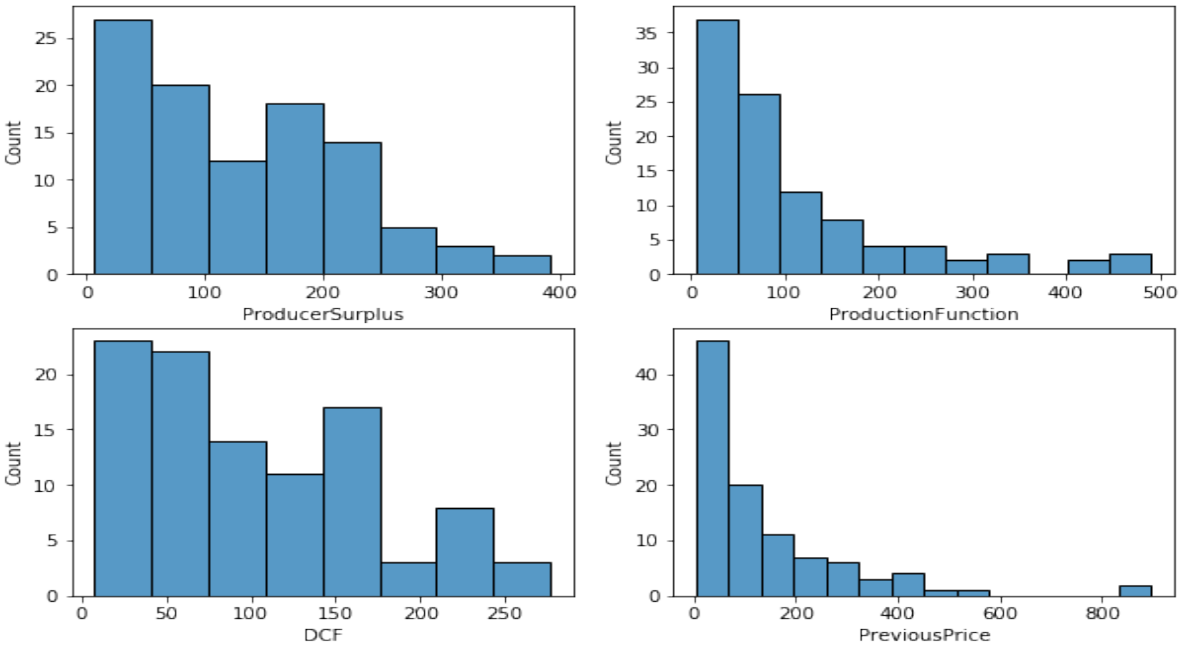
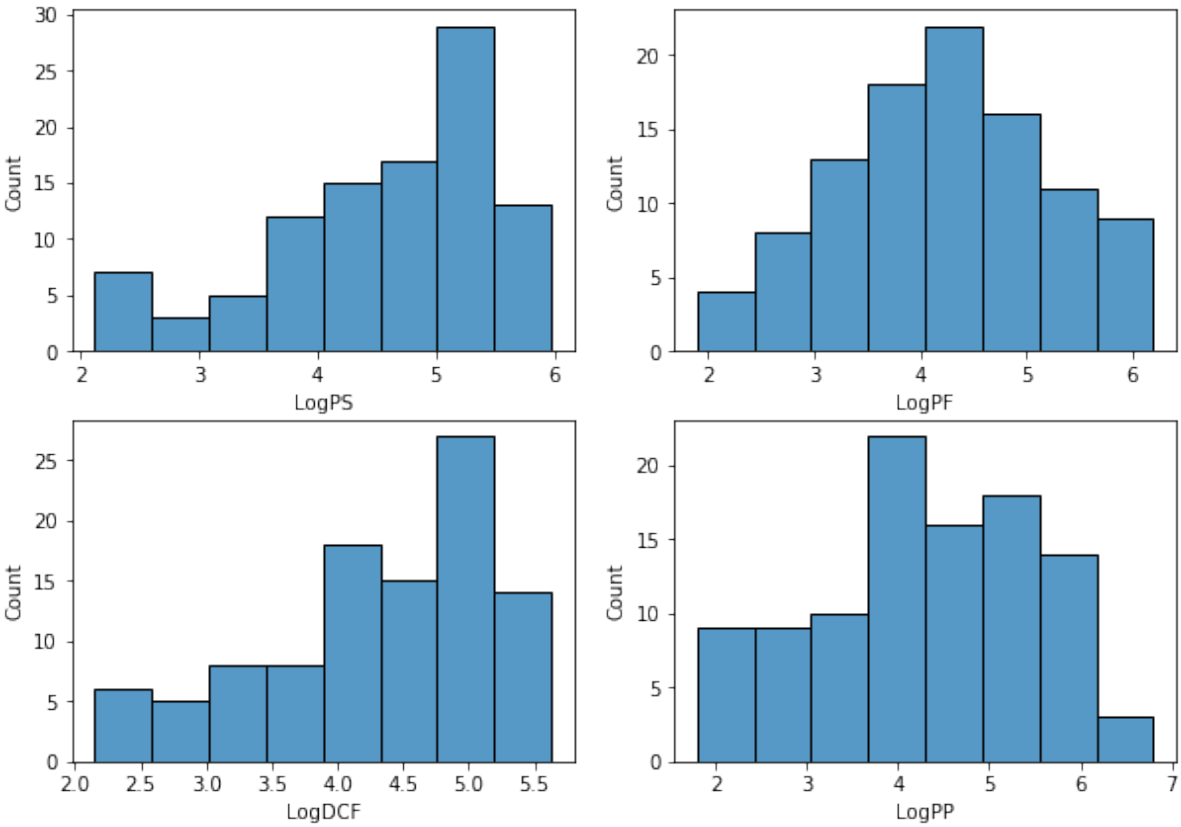

Count
ProducerSurplus
ProductionFunction
DCF
PreviousPrice

Fig-3


Count
LogPS
LogPF
LogDCF
LogPP

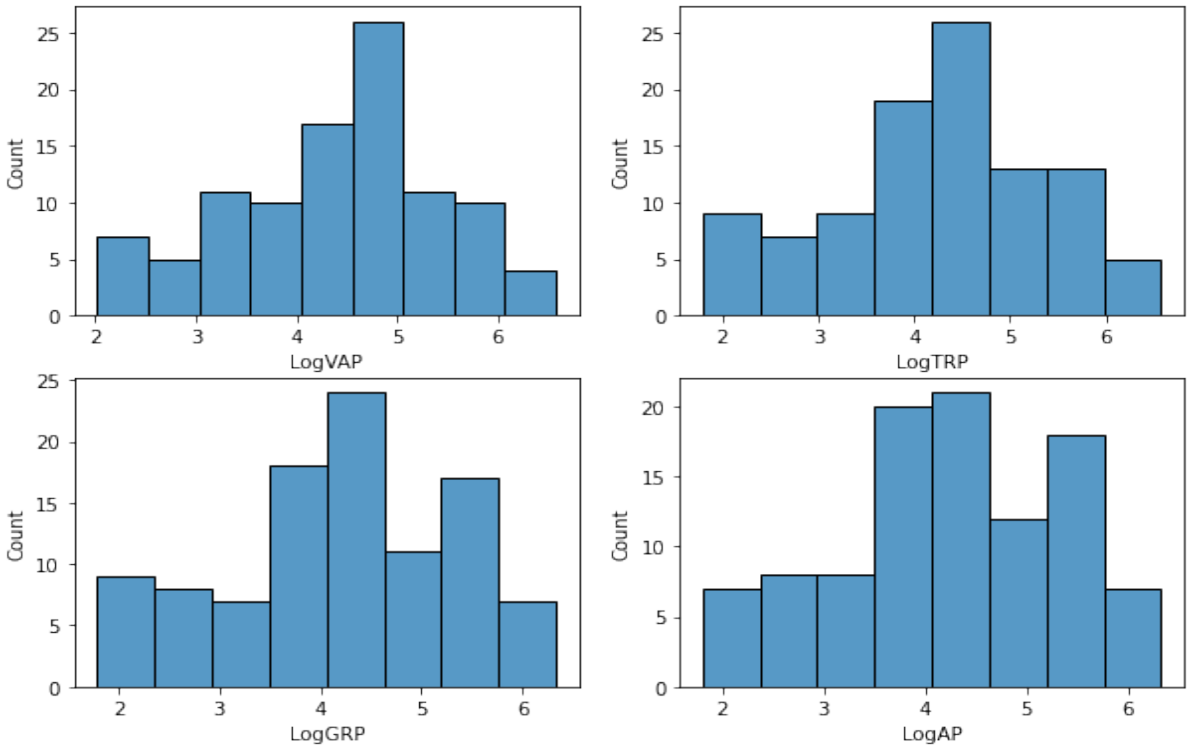


Fig-4

(iii)Scatter Plots: -

The scatter plots of the independent variables with respect to the response variables are produced in Fig-5 below here. The relation between the price input variables and output variables can be seen here following a linear pattern. The TRAI valuation metrics look having a realistic linear pattern whereas the last three variables LogPP_c(Previous Price log transformed and centred variable), LogTRP_c(Trai fixed price log transformed and centred) and GRP_c(Govt fixed price log transformed and centred) can be seen to have a perfect linear relation with the response variable LogAP(Winning price log transformed) which appears to be non-realistic. Intuitively, such variables can have overriding effect over other explanatory values and need further examination.

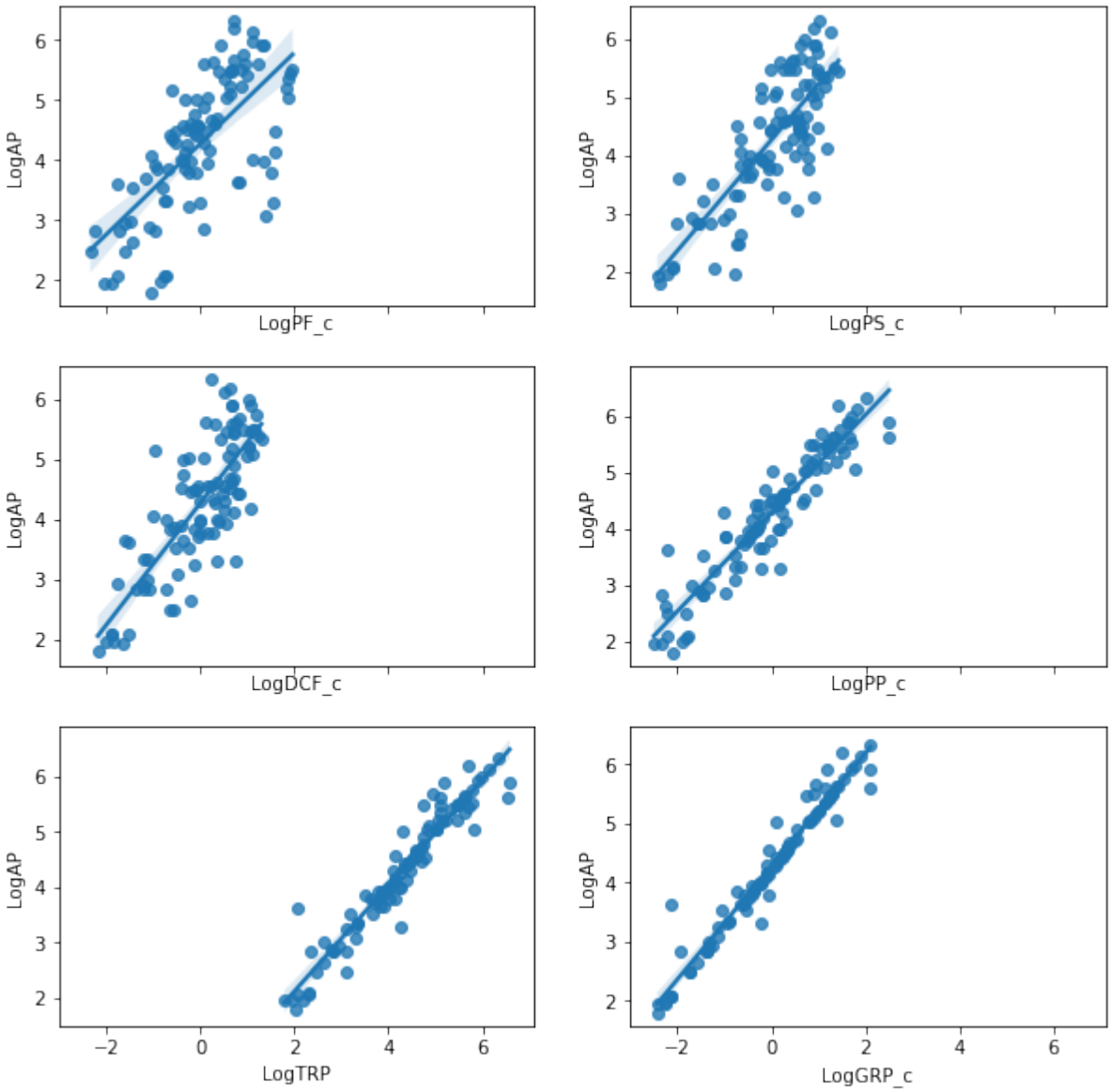


Fig-5

The Pearson correlations among the explanatory variables (shown in Fig-6 below) yield the following correlation matrix:

	LogGRP	LogTRP	LogPP
LogGRP	1.000000	0.991594	0.969514
LogTRP	0.991594	1.000000	0.970689
LogPP	0.969514	0.970689	1.000000

Fig-6

From the above matrix, it can be observed that their coefficients are unusually high which is indicative of the presence of multicollinearity among these variables. As can be seen from the auction process and explained in later paras, these values are more

often than not similar and highly collinear. Therefore, these variables can contribute towards poor modelling while performing regression and would be better if they are not considered in the modelling process.

6. **OLS Regression: -**

Regression analysis is a technique for modelling data. In the case of regression analysis, the focus is on the “conditional mean value" of a single dependent variable y corresponding to a given set of predictor (independent) variables x1, x2……. The conditional mean is a function that represents the mean of the dependent variable conditional on the dependence over the predictor variables. Ordinary Least Squares (OLS) is one of a type of regression analysis assuming linearity between the input and output variables. In other words, linear least squares represent the conditional mean function as a linear function of the predictor variables.

From the research question, the Null hypothesis can be stated as ‘there is no association between the response variable and the explanatory variables’. The OLS regression is done to explore if there is enough evidence in the data to reject the Null hypothesis. If not, Null Hypothesis cannot be ruled out.

1. As was explained above, all the three price variables namely, TraiRP, PP (Previous Price) and GovRP variable are found to have strong correlation among them and indicate the presence of multicollinearity problem. Hence these variables have been dropped from the regression. The remaining variables –the three TRAI metrics, the GSDP/capita, Population of each LSA, ARPU, Minutes of Use (MOU), Tele-density, number of mobile subscribers, Spectrum sold and the AGR were all considered as independent variables that can influence the Winning Price. It was found from the test that the Winning Price was not influenced by the telecom metrics ARPU, MOU and tele-density when these variables were included in the regression. It was also found from the regression test that the number of mobile subscribers was found to be confounding the population variable of the LSA. By calculating the Variance Inflation Factors (VIF) for presence of further multicollinearity helped to isolate the variable AGR and one of the TRAI metrics PS (Producer Surplus) to arrive at the final candidates for the regression. The OLS regression produced the following output as shown in Fig-7:

OLS Regression Results

Dep. Variable:	LogAP	R-squared:	0.885
Model:	OLS	Adj. R-squared:	0.875
Method:	Least Squares	F-statistic:	88.88
Date:	Wed, 07 Jul 2021	Prob (F-statistic):	6.73e-40
Time:	12:45:03	Log-Likelihood:	-45.373
No. Observations:	101	AIC:	108.7
Df Residuals:	92	BIC:	132.3
Df Model:	8		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	4.6314	0.107	43.113	0.000	4.418	4.845
C(Category)[T.B]	-0.5177	0.122	-4.228	0.000	-0.761	-0.275
C(Category)[T.C]	-0.9754	0.185	-5.273	0.000	-1.343	-0.608
C(Category)[T.M]	0.7812	0.162	4.831	0.000	0.460	1.102
LogDCF_c	0.3281	0.082	4.006	0.000	0.165	0.491
LogPF_c	-0.0447	0.063	-0.708	0.480	-0.170	0.081
LogSpectrum_c	-0.0630	0.048	-1.305	0.195	-0.159	0.033
GsdpPerCapita_c	0.2979	0.078	3.836	0.000	0.144	0.452
MobSubscribers_c	0.0168	0.003	5.787	0.000	0.011	0.023

Omnibus:	2.478	Durbin-Watson:	2.482
Prob(Omnibus):	0.290	Jarque-Bera (JB):	1.858
Skew:	-0.266	Prob(JB):	0.395
Kurtosis:	3.397	Cond. No.	152.

Warnings:[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Fig-7

The statistically significant variables are DCF, GSDP/Capita and Mobile Subscribers in a LSA. The Production Function (PF) and Spectrum are found to be statistically not significant in the regression. The final outcome of the regression can be represented as per the following correlation:

Log (Winning Price+1) = 4.63 + 0.33 X (log (DCF_c + 1) (DCF in Rs Crores/per MHz)
+ 0.30 X GSDP/Capita (In Rs Lakhs/per person)
+ 0.02 X Mobilesubscribers (in Millions)

(Note: - The Class of the LSA also has an impact which was represented by a dummy categorical variable, with the Class 'A' LSA representing the reference class. Its impact also can be seen in the regression model from the OLS output. Since the p-value of these dummy variables are also significant, there is an evidence of difference on the winning price (Response variable) among the 'Metro',' Class A', 'Class B' and 'Class C' classes, with Metro class having the highest coefficient. These coefficients will have to be used appropriately while predicting the price of future auction, by substituting the relevant coefficient in the above equation.)

Interpretation on what the regression throws out:

- In an LSA, for each unit of DCF change, the mean Winning Price changes by 0.33 times (DCF), both expressed in Log terms, when other variables are kept at their mean values.
- For each unit of GDSP/Capita, the mean Winning Price changes by 0.3 times (GSDP/Capita) the Winning Price term expressed in Log when other variables are kept at their mean value. The unit of GSDP/Capita is in Rs.Lakhs per person.
- For each unit of Mobile Subscribers, the mean Winning Price changes by 0.02 times (Mobile Subscribers) the Winning Price term expressed in Log when other variables are kept at their mean value.
- All variables have a positive linear association on the response variable.
- Out of the TRAI metrics, the DCF is only having an influence to determine the Winning Price.
- The GSDP/Capita effect is also equally influencing the Winning price.
- The mobile subscribers rather than the population of the LSA have the positive influence on the winning price.
- The 'goodness of fit' is 88 % meaning that this model can explain the variability of the response variable with the chosen explanatory variables to as much as 88 percentages.

Another point to note about the variables Trai fixed Price and Govt fixed Price. There is a strong correlation among these two independent variables which showed the presence of multicollinearity. If we conduct the regression of the Winning Price with either the TRAI fixed price or the Govt fixed price variable, the p-values show that they are statistically significant and with a 'Rsquared value' of the model as high as 96% indicating that the 96% of the variability of the auction price can be explained by either of these two variables. An explanation can be given if we observe the values of these two variables from the data set relating with the values of Winning price. The auction activity is judged from the bidding activity in an auction. Most often, the winning price is decided without any activity and sold at the reserve price only, especially in 2012, 2016 and 2021 auctions. This means that the Winning Price is same as the Govt Fixed RP. It may also be seen from the dataset, in many instances the TRAI Fixed RP becomes the final reserve price and de facto the 'Govt Fixed RP'. Therefore, these two metrics have undue influence on the final winning price on many

LSA s in the auction, especially where there is no auction activity. It is due to these factors, it was decided to drop the two variables from the regression.

7. **Regression Plots**: The ‘qq’ plot and the residual plots of the results of regression is given here:

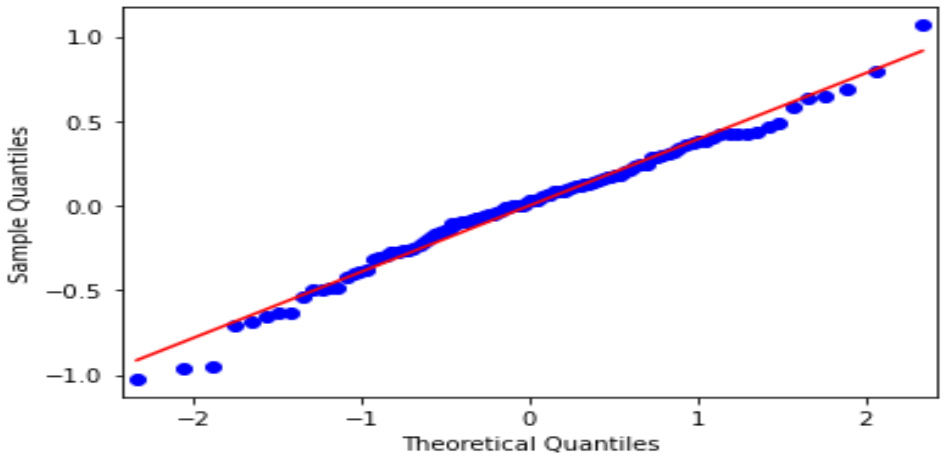


Fig-8

The QQ plot (Fig-8 above) shows that the linearity of the quantiles from the regression follows a normal distribution (except with the outliers) and the residual are Gaussian indicating that the confidence interval and other tests in the OLS regression are valid.

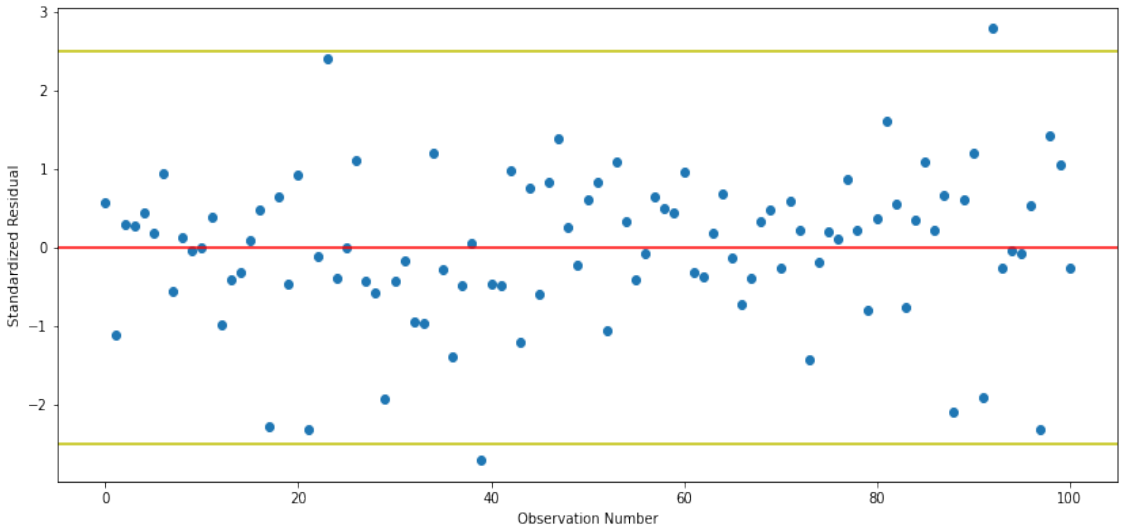


Fig-9

The residual plot with the residuals shown on the Y-axis and the fitted values shown on the X-axis is shown in Fig-9 above. The standardised residuals can be found from the plot that all are falling within the 2.5 standard deviation around the horizontal axis and the pattern shows that it randomly distributed. As there is no pattern formation in the residuals, it is a confirmation that there is no heteroscedasticity in the data fulfilling the assumption of the OLS regression that the error terms have a constant variance. Based on the above two plots, it can be concluded that the regression is valid.

8. **A comparison with the past research**: - It is interesting to see the comparison of the inference thrown by the Regression in this paper with that in the earlier paper where the dataset was limited to 80 records.

(i)The regression from the first paper:

Log (Winning Price) = 5.4287 + 0.3795 X Log (DCF in Rs Crores per MHz)
+ 0.4588 X GSDP/capita (in Rs Lakhs Per population)
+ 0.4476 X Log (Population in numbers)

(ii)From the regression in this paper:

Log (Winning Price+1) = 4.63 + 0.33 X (log(DCF_c+1) (DCF in Rs Crores/MHz)
+0.30 X GSDP/Capita (In Rs Lakhs/per person)
+0.02 X Mobilesubscribers (in Millions)

Augmenting the dataset from 80 records to 101 records has not brought substantial change in the model. The DCF metric continue to have similar influence on the Winning price in both the cases. Also the GSDP/Capita also maintains the similar level in both the cases. The only change is that the variable ‘Population of an LSA’ is superseded by the variable ‘number of mobile subscribers in an LSA’. This change sound logical as the population figure is already captured in the GSDP/Capita parameter and the Population variable would become a redundant variable. Further, the presence of the additional variable Mobile subscribers would add a telecom metric to the price determination model.

9. **Finding from the research**: -

A number of variables that are likely to influence the spectrum price were considered for this study. The parameters selected varied from estimated price information by the TRAI, the demographic information about an LSA, the telecom metrics associated with the LSA and the GDP/per capita of each of the LSA.

The estimate of Reserve price is done by TRAI using three different methodologies. In each of these methods, the TRAI itself uses financial, technical and other telecom performance indicators to come out with the different reserve price valuations for each of the LSA s. The influence of these indicators will be reflected indirectly in the price metrics that TRAI has evaluated. As the principle used are different in each of these three methodologies, the final estimated price also differs noticeably from one method to another and produces three diverse price metrics for the same LSA. That is the reason TRAI is taking the probabilistic average of all methods to finally recommend a single value for each LSA. This is represented by an additional variable used in the study which is named as 'Trai fixed RP'.

In the process of auctioning in DoT, this TRAI reserve price may be adopted as the final reserve price or might undergo some minor changes before it is declared as the Govt fixed reserve price. As there is no direct correlation from 'Trai fixed RP' to 'Govt fixed RP', the latter price is also used as another independent variable in study. Along with this variable, now there are six different price variables as independent variables for consideration.

In an auction, the spectrum price starts from the reserve price; depending on the auction activity, it can go up through the bidding and finally could be sold at a higher price than the reserve price. In certain other LSA s, the competitive bidding will be missing due to lack of demand of spectrum resulting in no upward movement of the price and finally will be sold at the reserve price itself. Most of the cases in this study are of the latter category where the spectrum is sold at the reserve price itself. It is seen from the data that in 70 % of the cases the spectrum is sold at the reserve price. Please see the table below (Fig-10). In other words, the response variable (winning price) becomes a copy of the 'Govt fixed RP' in which case, other determinants (input variables) can become extraneous in the association that we are trying to correlate. It is due to these reasons, these variables –Trai fixed RP and Govt fixed RP- cannot be considered as potential repressors to find the winning price.

Percentage of cases where the spectrum was sold at RP in terms of the number of LSA and in terms of quantum of spectrum						
	In terms of number of LSA			In terms of quantum of spectrum (in MHz)		
Year	* RP==WP	Total	Ratio (%)	* RP==WP	Total	Ratio (%)
In 2012	17	18	94.44	105	118.75	88.42
In 2014	11	22	50	191.67	319.27	60.03
In 2015	5	14	35.71	31.2	93.8	33.26
In 2016	13	15	86.67	126.4	173	73.06
In 2021	21	21	100	151.4	151.4	100
Total	67	90	73.33	605.67	856.22	70.74

*RP=Reserve Price
*WP=Winning Price

Fig-10

The above fact can be explained further through the box plots of these variables as below:

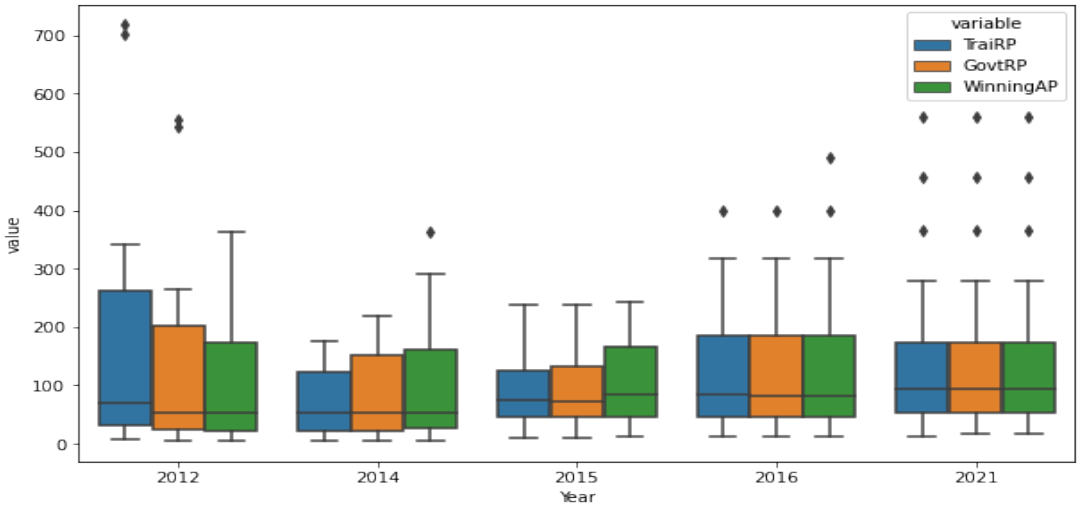


Fig-11

The boxplots of the reserve prices and the winning price shows the distribution of the data, which clearly demonstrates that the median values of all three variables remain ‘tied’ among them, noticeable across the different auction years. This means that the variance of the winning price get stagnated around the reserve price first fixed by the TRAI followed by the Govt.

The same data when plotted against different LSA categories can be visualised by the following plot that demonstrate the distribution. Some important observation from this plot is that the median value is aligned more or less in Class ‘A’, ‘B’ and ‘C’ circles whereas some minor variation(median value) can be seen in the Metro circles, on an average. Further, again on average terms, in Metro LSAs the reserve price fixed is slightly higher than the TRAI fixed price and the winning price is also higher indicating some activity in few cases of Metro LSAs. In Class ‘B’ LSA s, the Govt. fixed Reserve price has been brought down from the TRAI fixed price slightly, on an average. (Also note that the outlier in Class ‘B’ outclassing the Class ‘A’ price.)

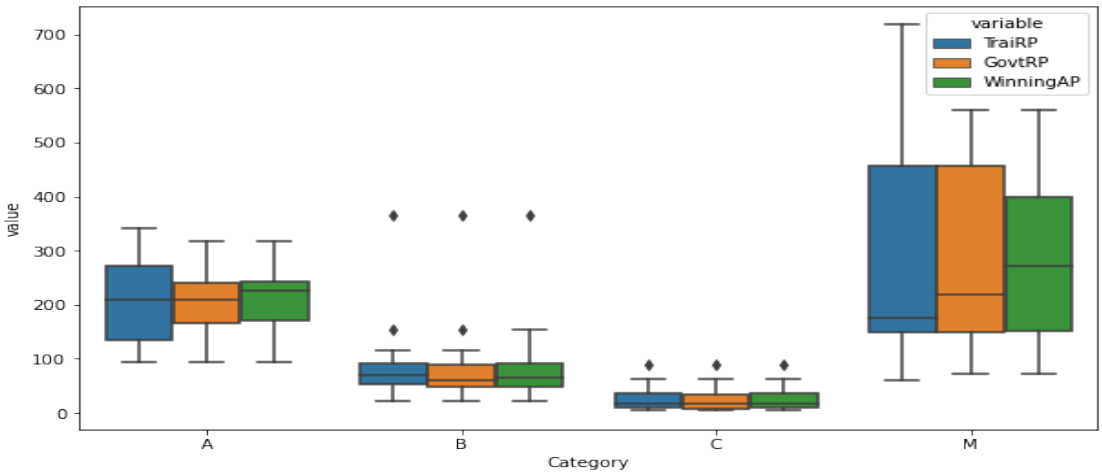


Fig-12

10. Conclusion: -

The two box plots down below (fig-13 and fig-14) show the representation of three price evaluation metrics estimated by TRAI in almost all the recommendations for auction for 1800 MHz spectrum. It clearly demonstrates the three metrics are not converging to a single price in any of the auction years as shown by the different median value. The LSA wise distribution also demonstrates that it is not bucking this trend in any of the LSA s. TRAI generally takes a probabilistic average of different methods to arrive at a final price.

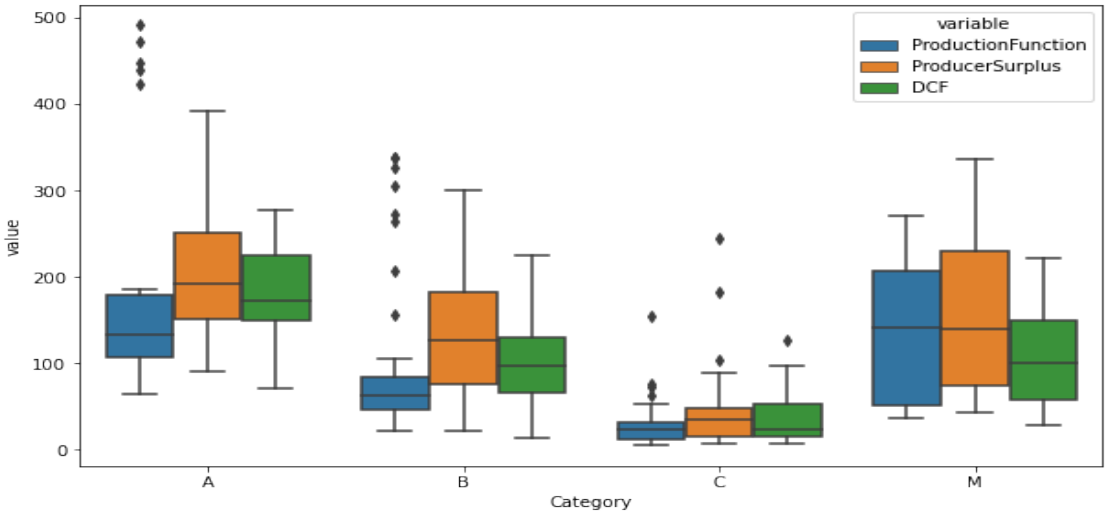


Fig-13

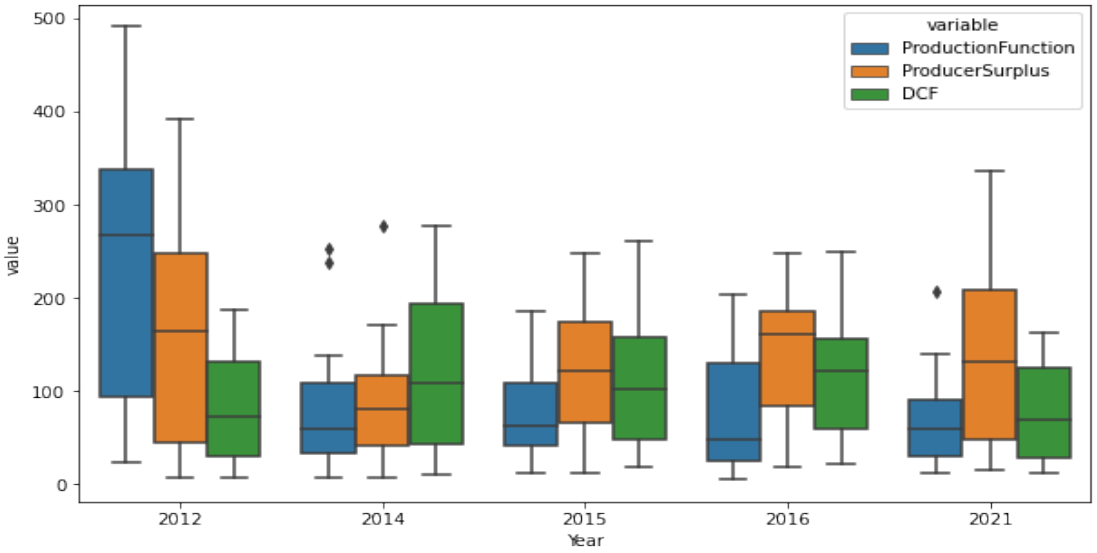


Fig-14

When the three prices estimated by different philosophies are grossly divergent, taking average of them may yield an inefficacious price that is faithful to none. What is suggested is that TRAI can discard the average calculation and follow the regression procedure in this paper (*edit-author not referring to the regression process used by TRAI to equate the last year price to this year price*) by assimilating the past data on these metrics derived by TRAI for various years and the latest estimated valuation metrics. The unique value so arrived from the regression output

would be the new reserve price for each LSA which in effect will be a true representative of past and present market information. While this line of action can make the price manoeuvring exercises expendable in the Ministry, DoT can draw succour by landing on the ideal reserve price that would propel it to a speedy road to auction and of improving the possibility of finding a true market price of spectrum.

11. Reference.

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12. Abbreviations: -

AGR-Aggregated Gross Revenue.

AP- Auction Price.

DCF- Discounted Cash Flow.

DoT- Department of Telecom.

GOVT- Government.

GSDP- Gross State Domestic Product.

LSA- Licensed Service Area.

OLS- Ordinary Least Square

PF- Production Function.

PP- Previous Auction Price.

PS- Producer Surplus.

RP- Reserve Price.

TRAI- Telecom Regulatory Authority of India.

WPC- Wireless Planning and Coordination.

Editorial Note: This is the second version of the original paper published on 18-05-2021 in which the auctioned price of spectrum for North East LSA in 2021 auction has been printed incorrectly as Rs.19 Crores. The actual auctioned price is Rs.17 Crores. The full document accordingly has been revised and designated as ver.2.